Explicativity, Corroboration, And The Relative Odds Of Hypotheses

PLAI Reading Group, Oct. 8th 2019

Vaden Masrani

The Probability Wars

- $P(A) \in [0,1]$
- $P(\overline{A}) = 1 - P(A)$
- $P(A \ or \ B) = P(A) + P(B) - P(A \ and \ B)$
- $P(A \mid B) = P(A \ and \ B)/P(B)$
- $P(A \ and \ B) = P(A \mid B)P(B) = P(B \mid A)P(A)$

The Probability Axioms

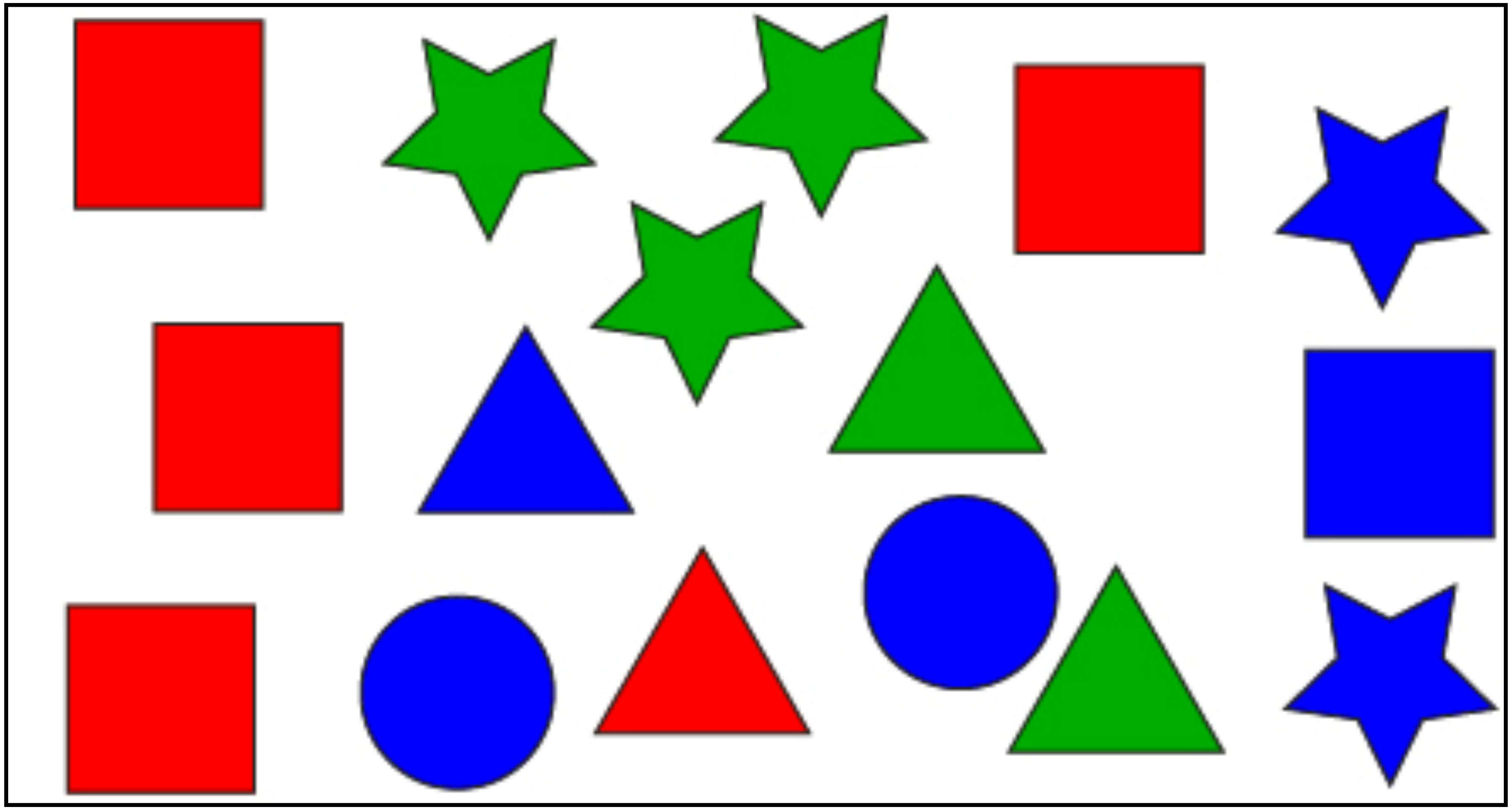
*“There are several schools of thought regarding the interpretation of probabilities, **none of them without flaws, internal contradictions, or paradoxes.**”*

–de Elía, Ramón; Laprise, René (2005). "Diversity in interpretations of probability: implications for weather forecasting"

Chance

Chance

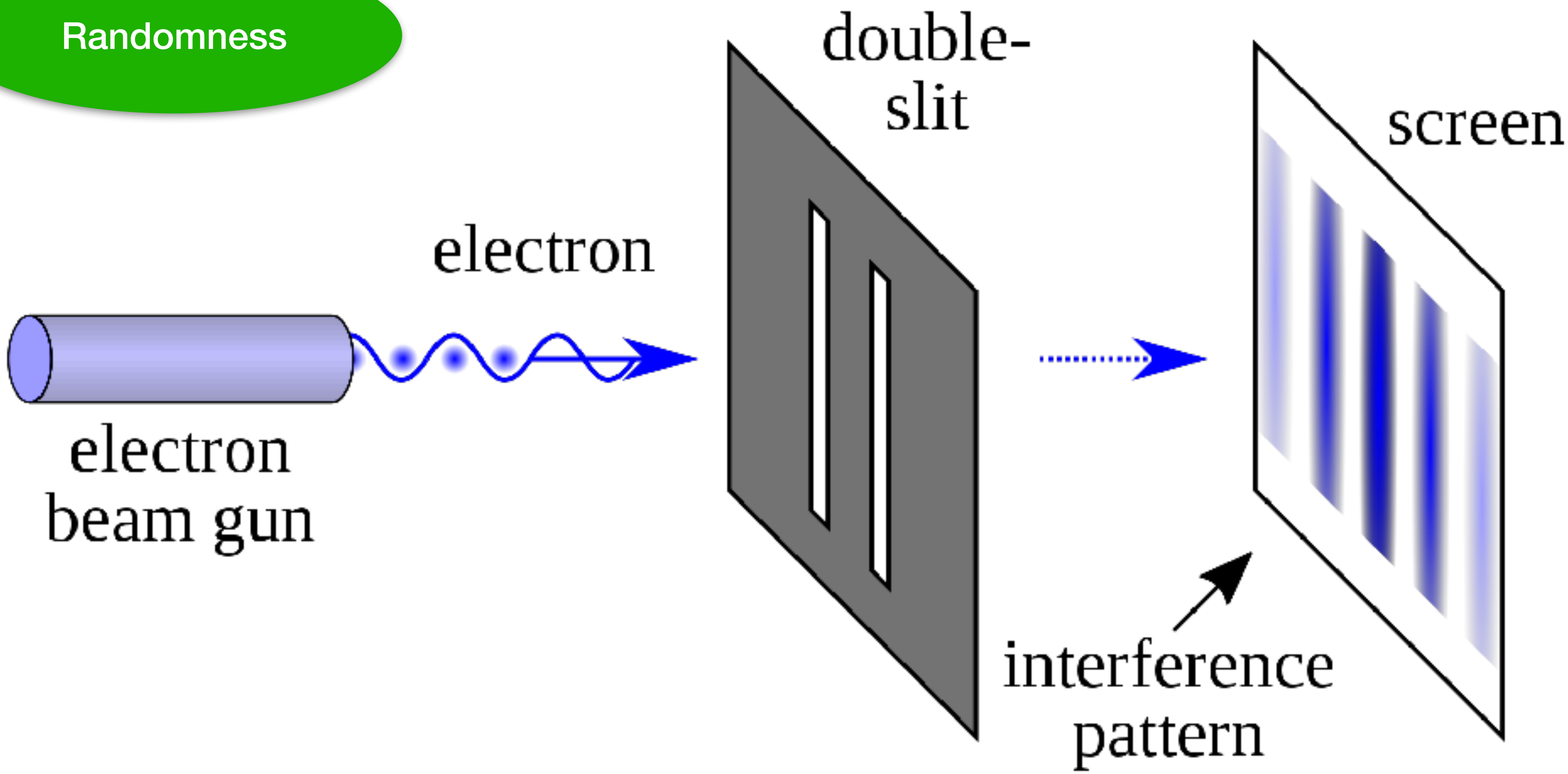
Frequency



Chance

Frequency

Randomness



Chance

Frequency

Randomness

Uncertainty

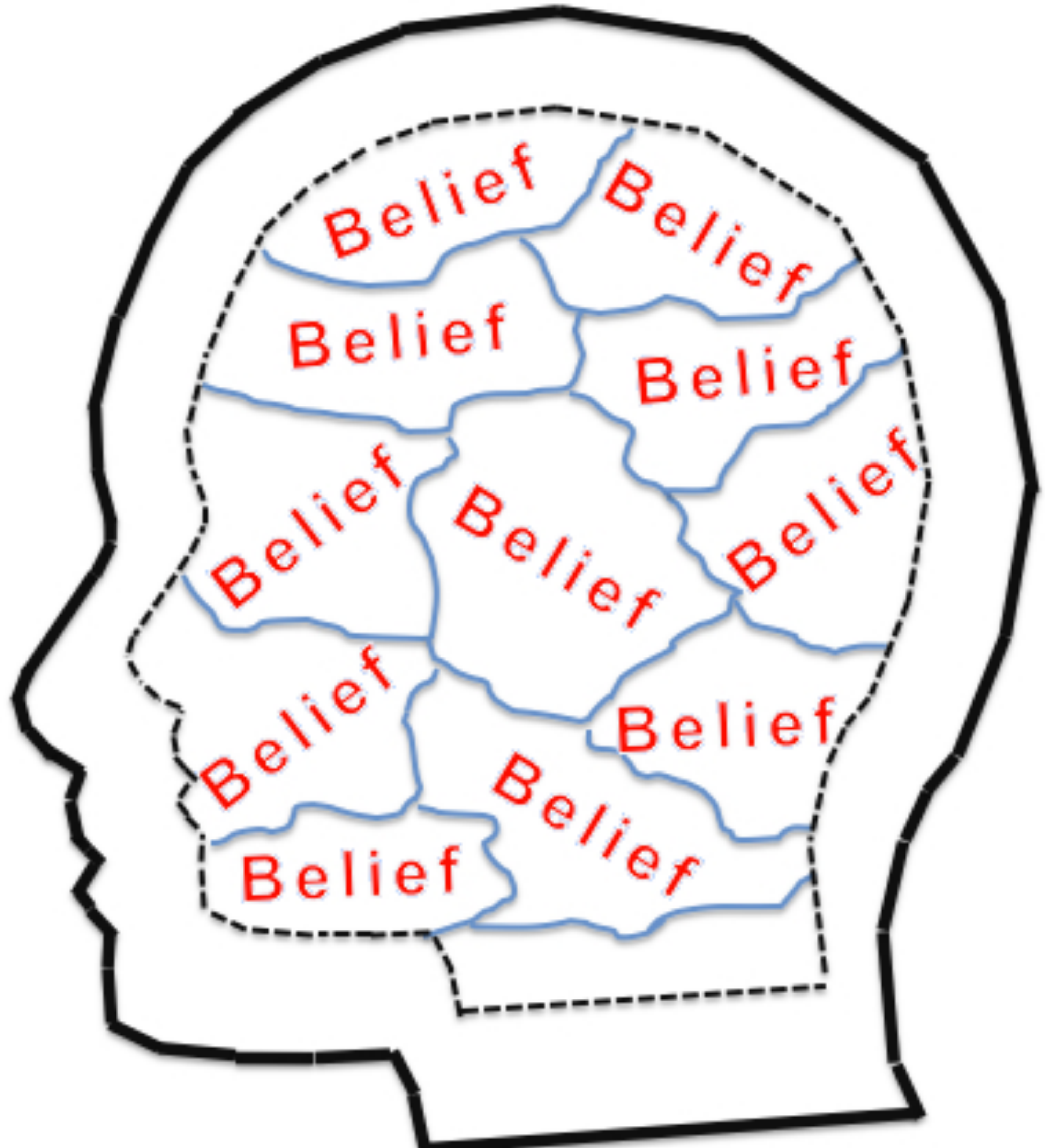
Chance

Frequency

Randomness

Uncertainty

Belief



Chance

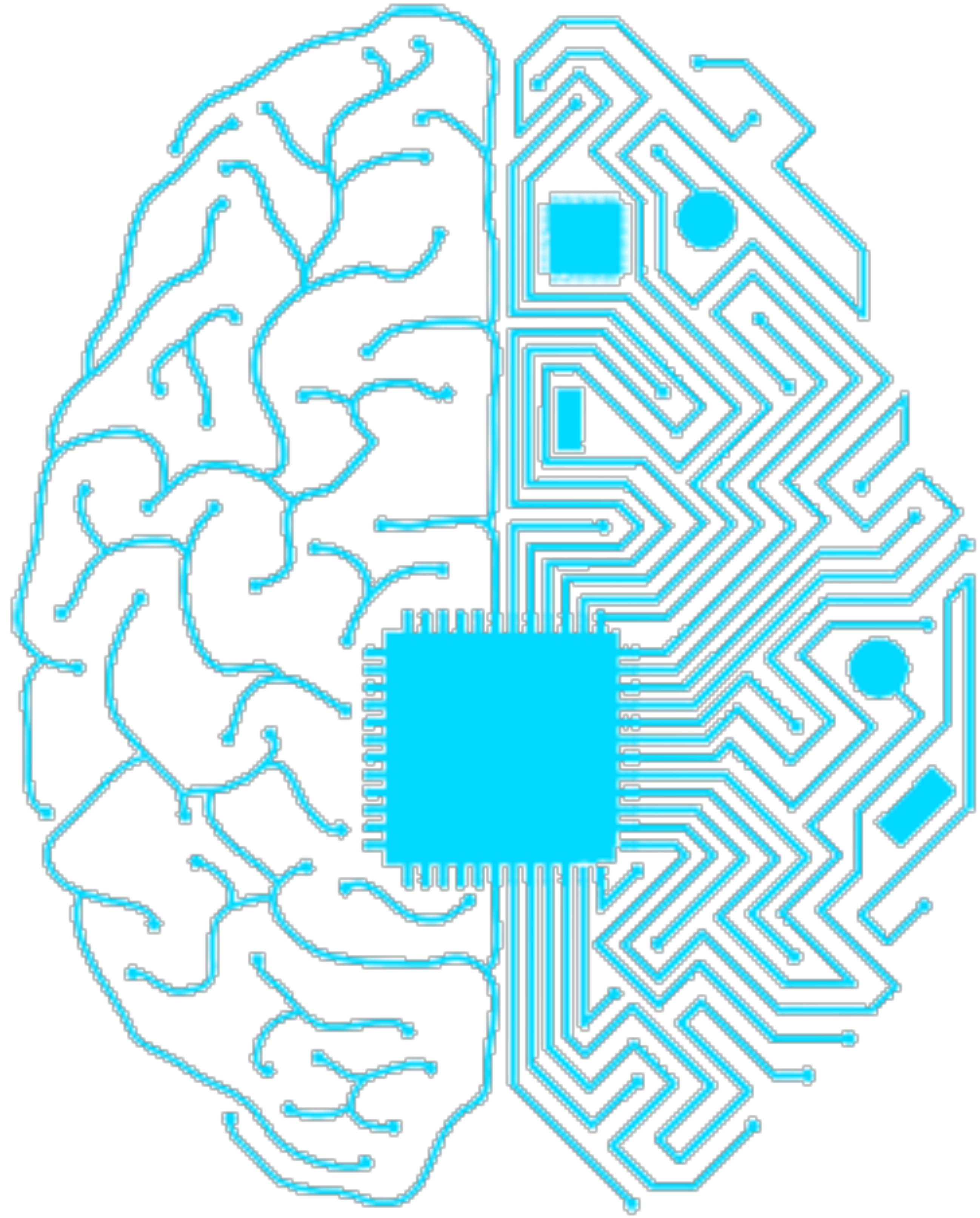
Frequency

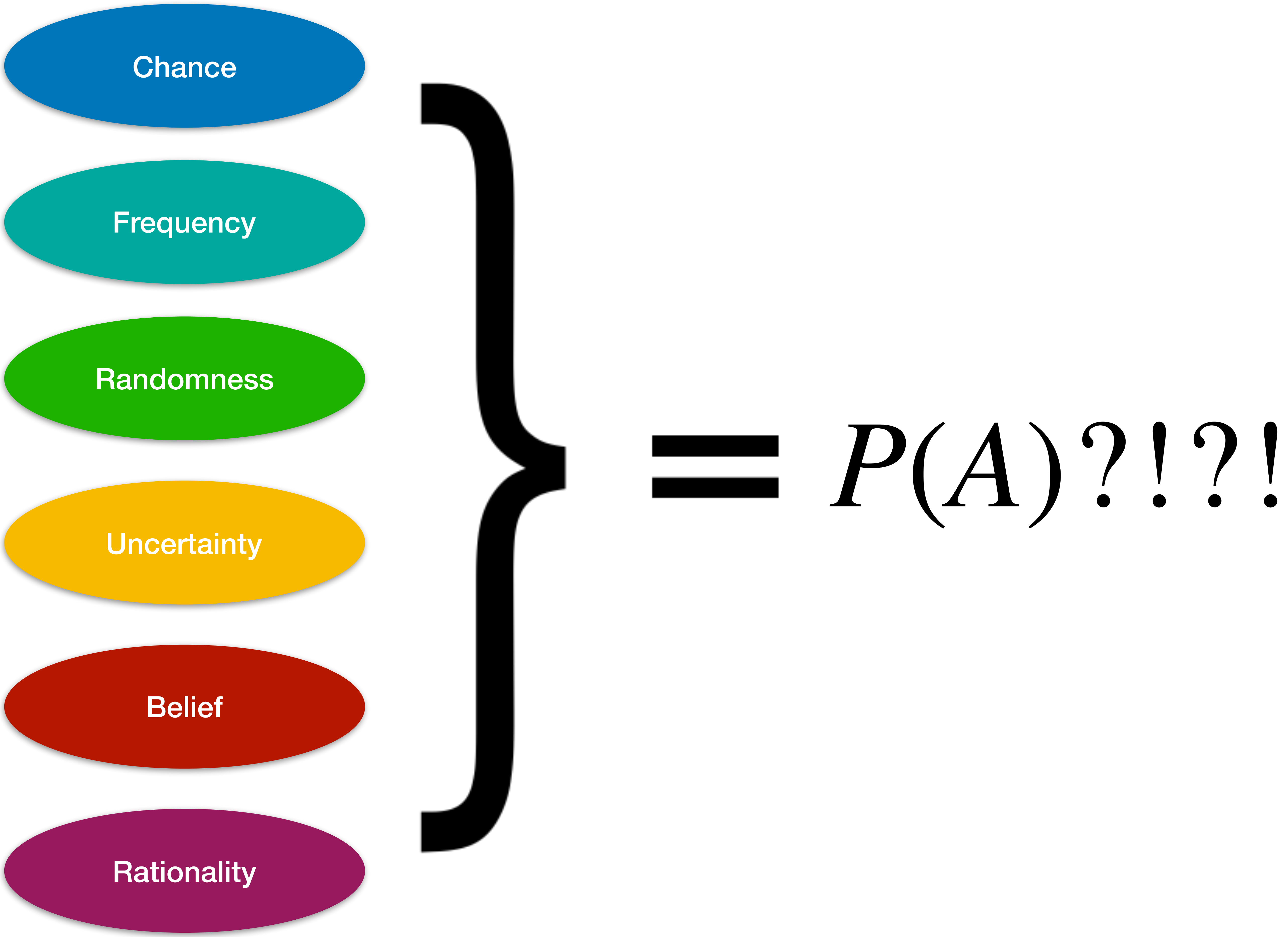
Randomness

Uncertainty

Belief

Rationality




Chance
Frequency
Randomness
Uncertainty
Belief
Rationality
= P(A)?!?!

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↑

Kolmogorov Axioms

1. *(Definition: Probability Triple)* Let Ω be a set, $\mathscr{F}$ be a σ-algebra on Ω, and $\mathbb{P}$ be a set function.
2. *(Axiom: Normalization)* $\mathbb{P}(\Omega) = 1$.
3. *(Axiom: Non-negativity)* $\mathbb{P}(A) \geq 0$ for all $A \in \mathscr{F}$.
4. *(Axiom: Countable Additivity)*: $\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$ for all countable collections of disjoint sets $A_i \in \mathscr{F}$.

Terenin, Alexander and Draper, David (2015), "*Cox's Theorem and the Jaynesian Interpretation of Probability*"

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Cox's Theorem

I. States of uncertainty are represented by real numbers.

II. Qualitative correspondence with common sense.

(a) If the truth value of a proposition increases, its probability must also increase.

(b) In the limit, small changes in propositions must yield small changes in probabilities.

III. Consistency with true-false logic.

(a) Probabilities that depend on multiple propositions cannot depend on the order in which they are presented.

(b) All known propositions must be used in reasoning – nothing can be arbitrarily ignored.

(c) If, in two settings, the propositions known to be true are identical, the probabilities must be as well.

Terenin, Alexander and Draper, David (2015), "*Cox's Theorem and the Jaynesian Interpretation of Probability*"

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Lead to

Kolmogorov Axioms

- ***Frequentist*** interpretation
- A, B are ***events***
- Probability is ***objective***
- P(A) is the ***relative frequency*** of A

Example:

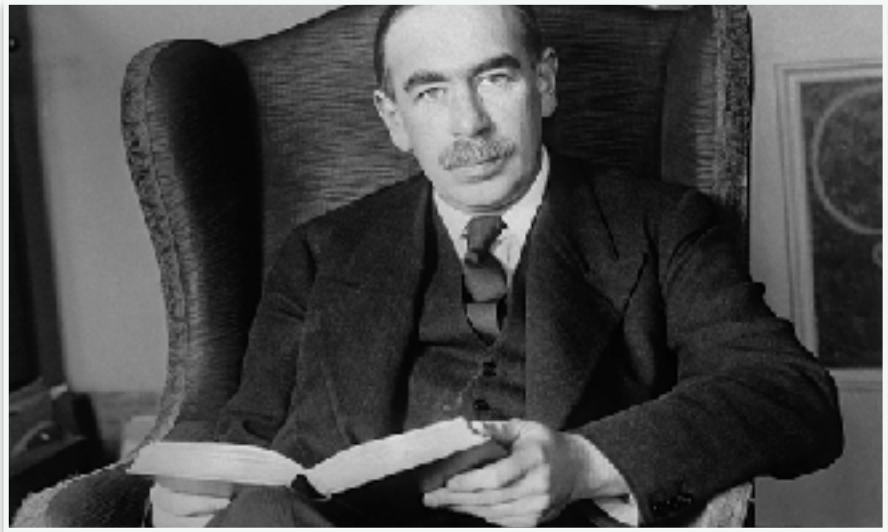
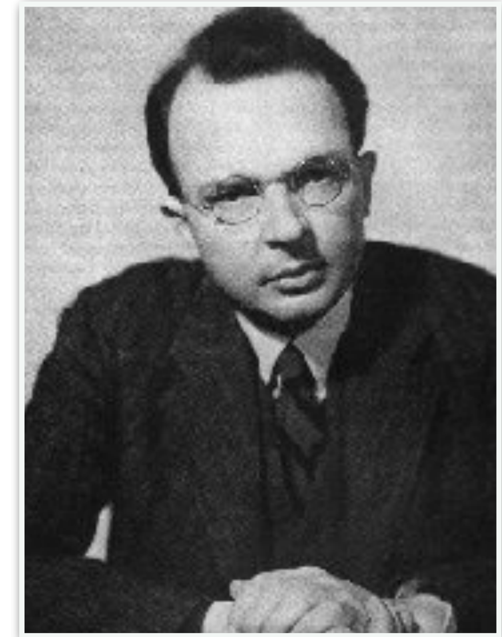
- A = Today is Monday
- P(A) = 1/7

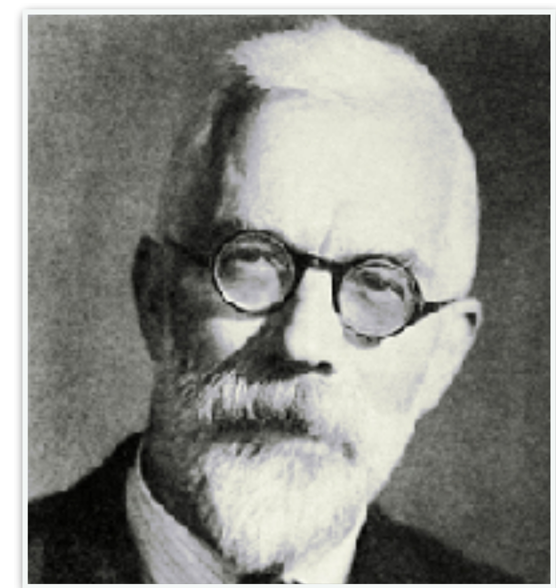
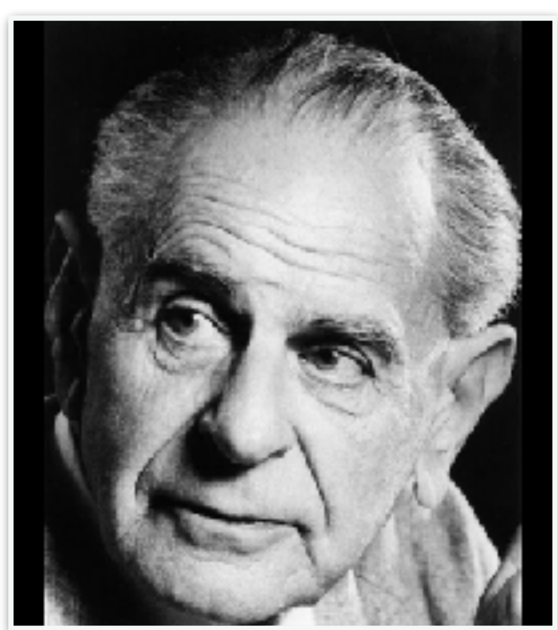
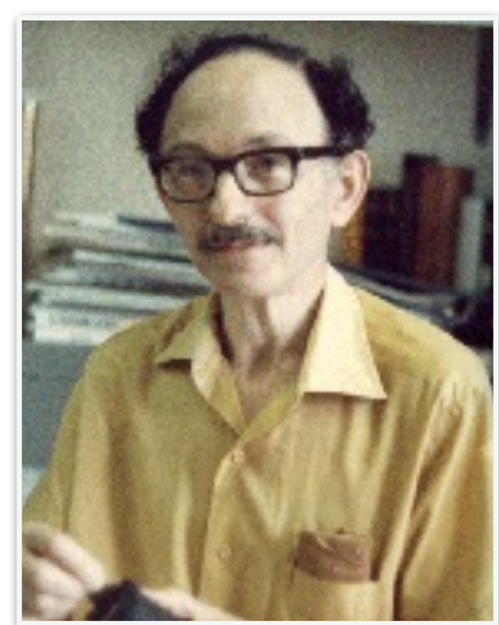
Cox Theorem

- ***Bayesian*** interpretation
- A, B are ***sentences***
- Probability is ***subjective***
- P(A) is the ***belief an agent has*** in A

Example:

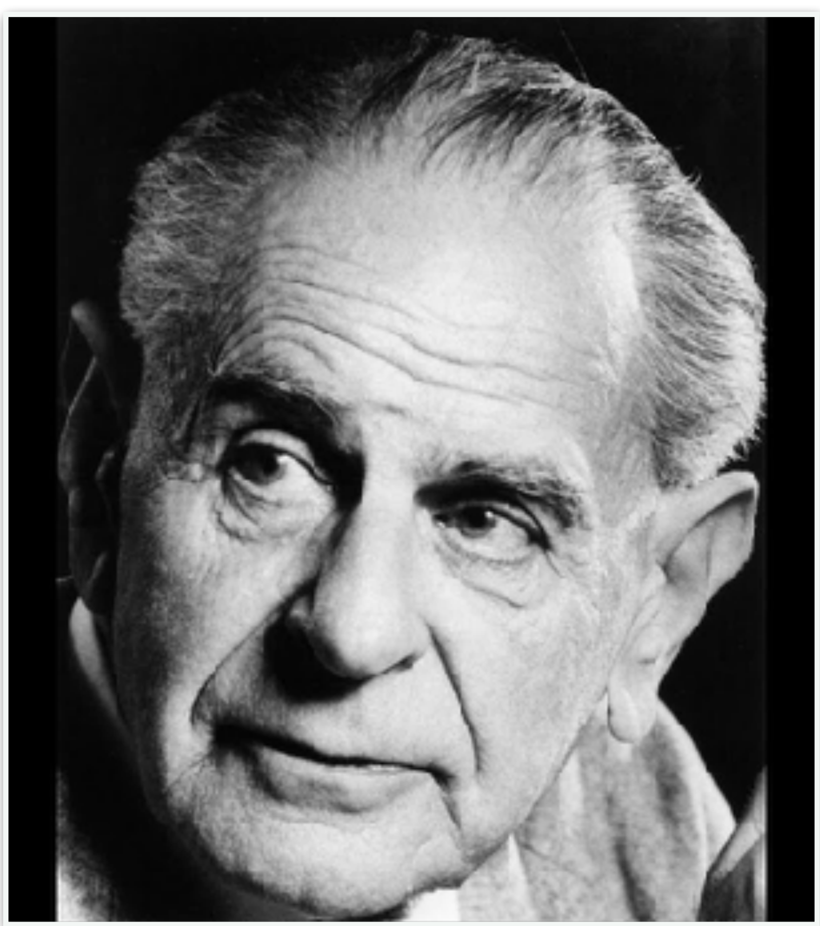
- A = "*Today is Monday*"
- P(A) = anything

“'Momma do you think it's proper; how did you ***react to Poppa***?'
- Popular song. ”

– *Good, Explicativity Corroboration And The Relative Odds Of Hypotheses*



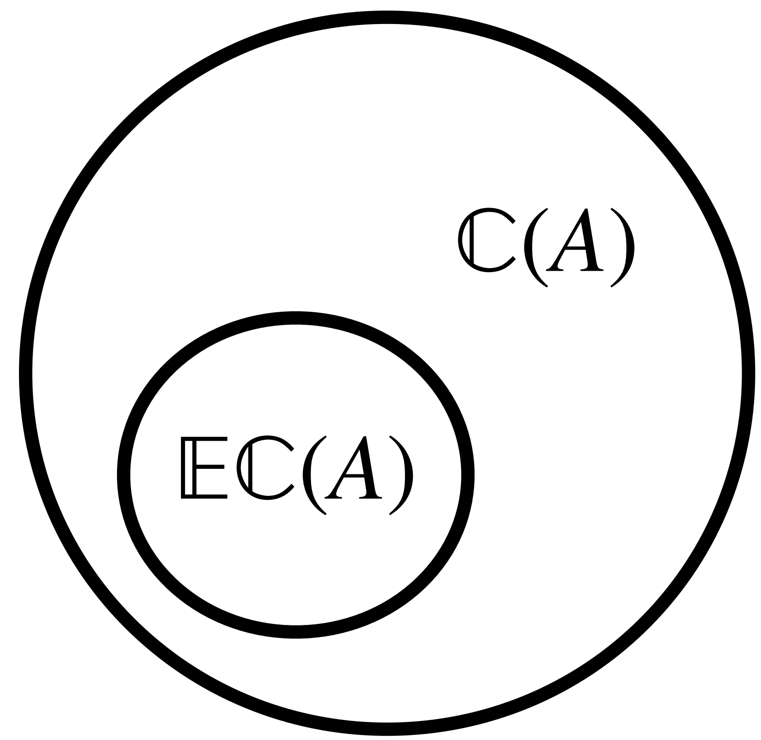
Content

- The ***content*** of a statement A, denoted $\mathbb{C}(A)$, is ***the class of all logical consequences*** of A

> - A = "*Today is Monday*"
> - $\mathbb{C}(A)$ = *{"Today is not Tuesday", "Today is not Wednesday"...}*

- The ***empirical content***, denoted $\mathbb{E}\mathbb{C}(A)$, is ***the class of all empirical statements*** logically entailed by a statement A

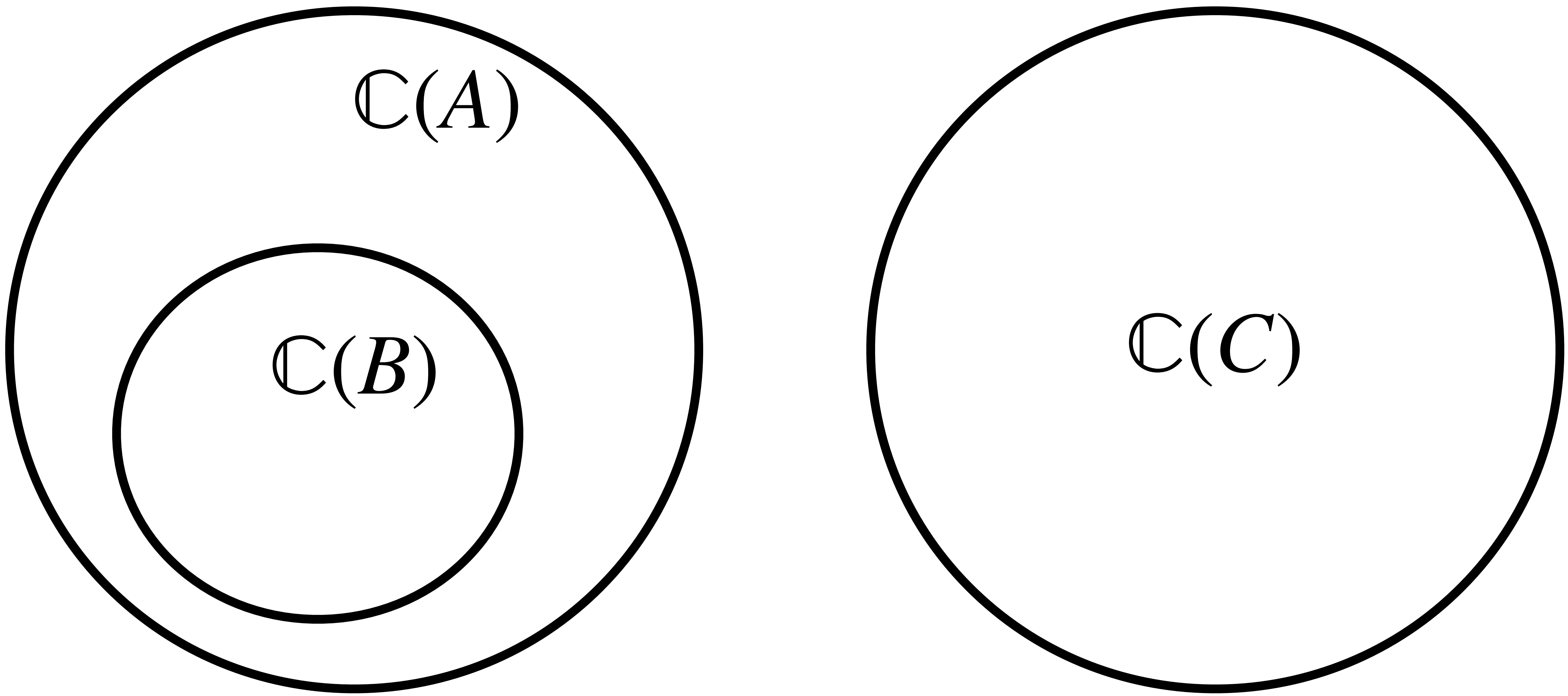
> - A = "*All ravens are black*"
> - $\mathbb{E}\mathbb{C}(A)$ = *{"There is not a white raven in Vancouver", "There is not a green raven in Calgary"...}*



- The empirical content forms the class of ***potential falsifiers*** of a statement A

- The content of two statements can be compared by the ***class/subclass relation***

> - A = "*All orbits of celestial bodies are circles*"
> - B = "*All orbits of planets are circles*"
> - C = "*DNA is structured as a double helix*"



More facts about content

- ***Metaphysical statements*** have logical content but no empirical content

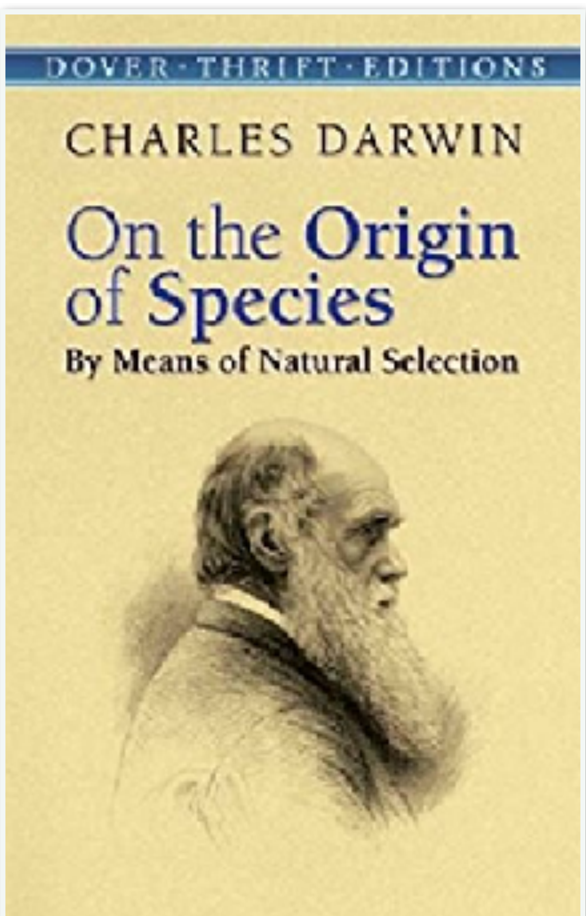
  - A = "*The arc of history bends towards justice*"
  - $\mathbb{C}(A)$ = {"The future will be more just than the past", ... }
  - $\mathbb{E}\mathbb{C}(A) = \{\}$

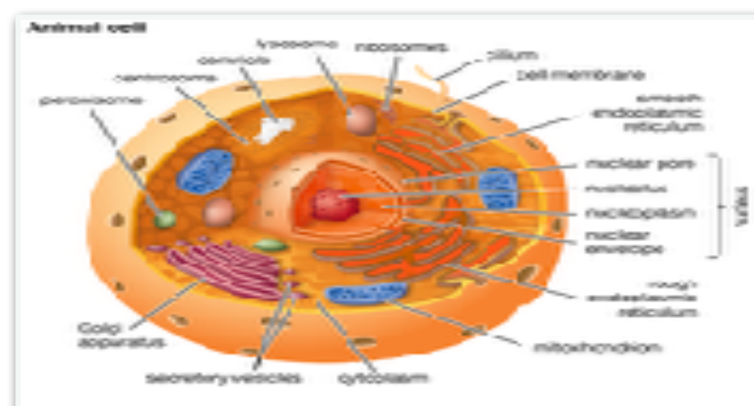
- ***Tautological statements*** have no / 0 content
- ***Contradictory statements*** have full / 1 content
  - And are therefore falsified by any empirical statement
- ***False statements*** can still have truth content

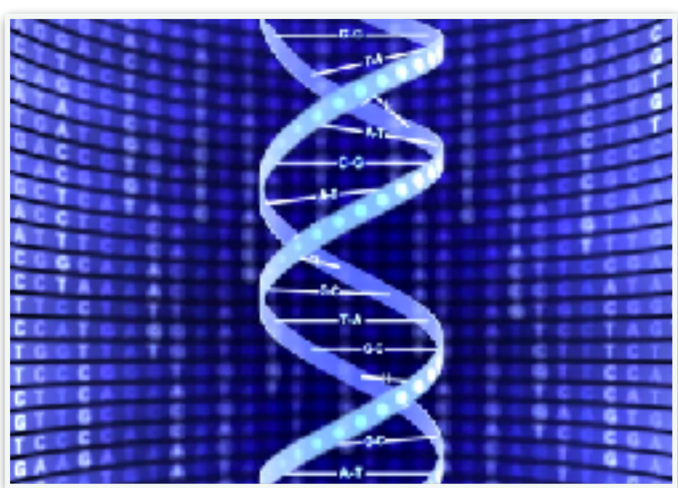
  - A = "*It is Monday 2001*"
  - $\mathbb{C}(A)$ = *{"It is not Tuesday 2001", ... }*

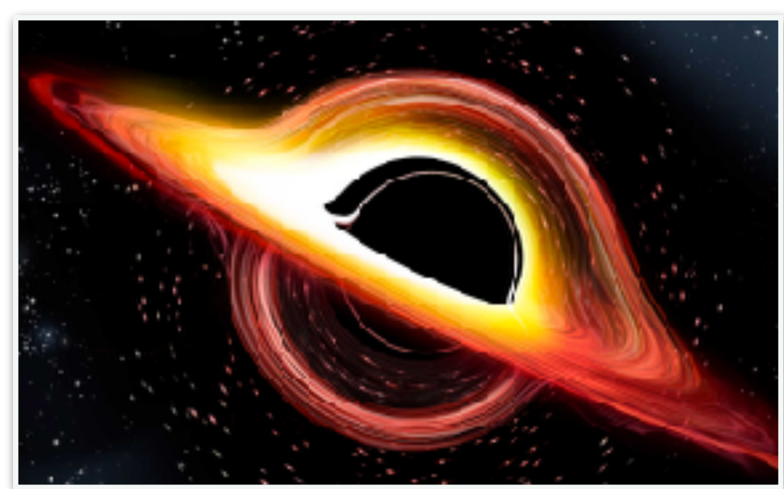
More facts about content

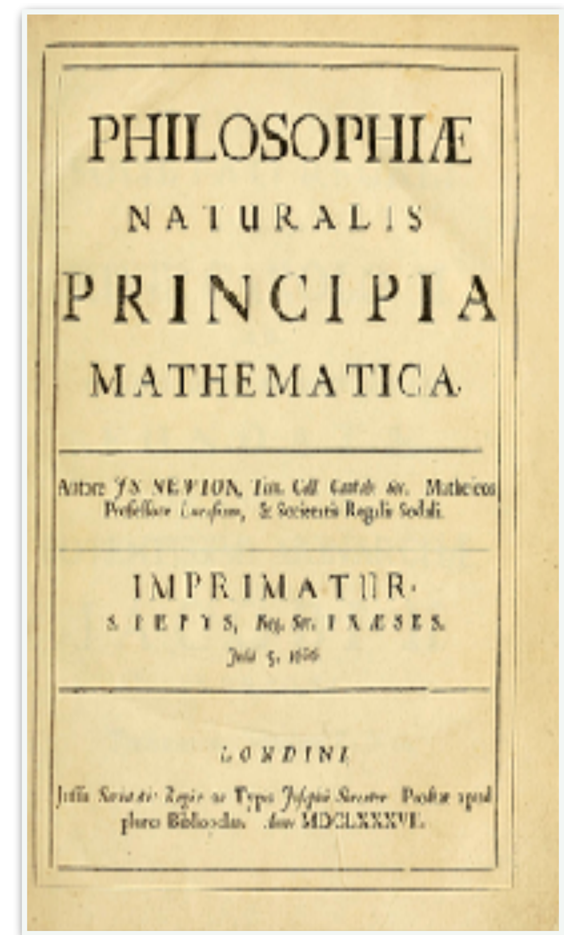
- In science we prize ***bold theories***
  - By "bold", we mean theories with ***high empirical content*** which therefore take ***great risks*** of being falsified











Content and Probability

- Consider two statements:
  - A = "It is Monday"   B = "It is raining"
- We have:
  - $\mathbb{C}(A) \leq \mathbb{C}(A \text{ and } B) \geq \mathbb{C}(B)$
- We also have:
  - $P(A) \geq P(A \text{ and } B) \leq P(B)$
- ***As the content of a statement grows, the probability decreases, and vice-versa***

$\mathbb{C}(tautology) = 0$	$P(tautology) = 1$
$\mathbb{C}(contradiction) = 1$	$P(contradiction) = 0$

“Those who identify confirmation with probability must believe that a high degree of probability is desirable. They implicitly accept the rule: 'Always choose the most probable hypothesis!' ”

–Popper, Conjectures and Refutations

“Now it can be easily shown that this rule is equivalent to the following rule: 'Always choose the hypothesis which goes as little beyond the evidence as possible!'”

–Popper, Conjectures and Refutations

“And this, in turn, can be shown to be equivalent to 'Always accept the hypothesis with the lowest content!' (within the limits of your task)”

–Popper, Conjectures and Refutations

*“This may sound paradoxical to some people. But if high probability were an aim of science, then scientists should say as little as possible, and preferably utter tautologies only. But their aim is to 'advance' science, that is to **add** to its content. Yet this means lowering its probability. ”*

–Popper, Conjectures and Refutations

Summary of background material

- There is ***no consensus on the interpretation of the probability calculus***

- This paper is primarily I.J. Good's response to Karl Popper, and a defense of the subjectivist view of probability

- Popper's critique is informed by the logical concept of *content*

- Now on to the paper...

Paper structure

I. Good's philosophy of probability

II. Complexity

III. The Probabilities And The Relative Odds Of Theories

IV. Weight of evidence

V. Explicativity And Predictivity

VI. Induction

VII. Testability Significance Tests And Checkability

I: Good's philosophy of probability

“Carnap ... said that what I call '***subjective probability***' would be better called '***rational credibility***” ...

– Good, 1975

*“...... but since **it depends on the person ... making the judgment** ... I think 'subjective' or 'personal' ... is a better term."*

– Good, 1975

“We all have to make subjective probability judgments but the person who recognizes this clearly enough is prepared to constrain his judgments so that they tend to satisfy a certain set of axioms."

– Good, 1975

“*In my opinion one might just as well assume that the physical probabilities exist as well as the subjective ones.*"

– Good, 1975

II: Complexity

Complexity

- Goal: Find a mathematical formula for the complexity of a proposition

- Attempt 1:

    - Equate complexity with Shannon information.

    - For a proposition H,

$$complexity(H) = I(H) = -\log p(H)$$

Complexity

- Problem with attempt 1

  - Let H = "*one equals two*"
  - P(H) = 0
  - complexity(H) = -log(0) = inf.

- Is the statement "*1 = 0*" infinitely complex? That doesn't seem right...

Complexity

- Attempt 2:
  - Still use $complexity(H) = I(H) = -\log p(H)$
  - But "frequentize" P(H) by defining P(H) as the probability the expression would occur in some language up to di-word frequencies
  - Then $p($"one equals two"$) \neq 0$

Complexity

- Still seems unsatisfactory...
    - By this definition the statement A = "Vaden exercised" is highly complex b/c the diword probability is low
    - "*Apparently then there is no clear-cut relationship between probability and complexity when negations or disjunctions are allowed*"

Complexity: Popper's solution

- Link the concept of complexity to ***compositionality***

- Greater compositionality -> greater complexity

  - **A =** "*Swans in Latvia are white **and** swans in Lithuania are white **and** no swans in Washington are black **and ...**"*
  - **B =** "*All swans are white*"

Complexity: Popper's solution

- Link concept of simplicity to ***universality***

- Universal statements logical imply less universal statements

- Thus they have greater empirical content (and are therefore more testable)

  - **A =** "*Swans in Latvia are white* ***and*** *swans in Lithuania are white* ***and*** *no swans in Washington are black* ***and …***"
  - **B =** "*All swans are white*"

“Above all, our theory explains why simplicity is so highly desirable. To understand this there is no need for us to assume a ‘principle of economy of thought’ or anything of the kind.

–Popper, Logic of Scientific Discovery

"Simple statements, if knowledge is our object, are to be prized more highly than less simple ones because ***they tell us more; because their empirical content is greater; and because they are better testable."***

–Popper, Logic of Scientific Discovery

Sections III - V and VII

III: *Probabilities And The Relative Odds Of Theories,*
IV: *Weight of evidence explicates corroboration*
V: *Explicativity And Predictivity*
VII: *Testability, Significance Tests, and Checkability*

III - V, VII

- Good wants to rest the foundations of science on ***subjective probability***
- Therefore he wants to use subjective probability to...
  - ... adjudicate between competing hypothesis (Ch. III)
  - ... quantify how evidence 'corroborates' a hypothesis (Ch. IV)
  - ... define the explanatory power of a hypothesis (Ch. V)
  - ... explain how knowledge is created (Ch. VI)
  - ... define the testability of a hypothesis (Ch. VII)
- This is in direct opposition to Popper's view of science
- Let's compare their philosophies.

Good	Popper
Tool: ***Probability***	Tool: ***Content***
Desires ***highly probable*** theories	Desires ***high content*** theories
A highly corroborated theory is one which is ***best supported by the evidence***	A highly corroborated theory is one which has ***resisted repeated attempts at falsification***
Truth = Certainty	Truth ≠ Certainty
Subjective view of science	***Objective*** view of science

Good

- Theories ***can not be shown to be true or false,*** only more or less probable
- Knowledge is ***subjective***
- Knowledge consists of ***highly probable theories***
- Knowledge is produced by ***probabilistic induction***

Popper

- Theories can not be shown to be true, ***but can be shown to be false.***
- Knowledge is ***objective***
- Knowledge consists of ***bold theories with high empirical content***
- Knowledge is produced by ***conjecture and criticism***

“We need to take into account all other evidence and also the relative initial odds of (General Relativity as compared with Newtonian physics). In my opinion the relative initial odds in favor of Newtonian physics do not exceed 10000 whereas the factor against it is far greater than 10000 ..."

– Good, 1975

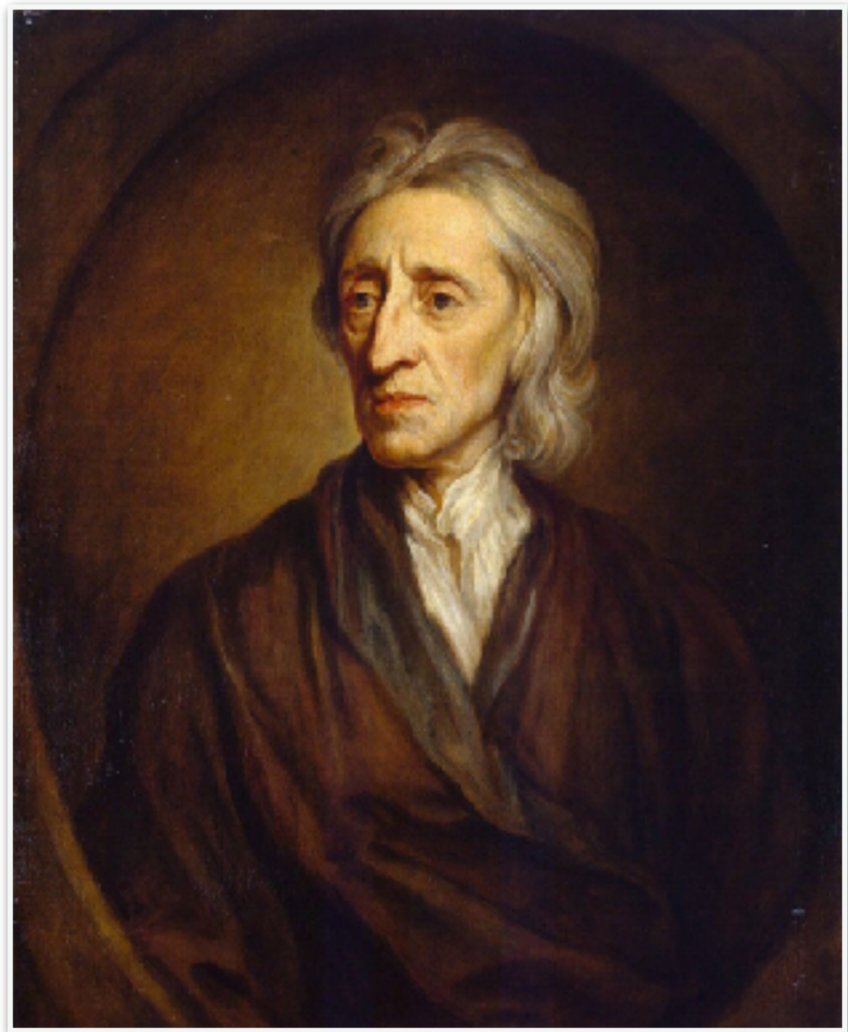
“Therefore, in my opinion, General Relativity is very heavy odds on as compared with Newtonian physics. I believe most physicists would agree with me once they understood what I am saying."

– Good, 1975

VI: The problem of Induction

The Empiricists

Francis Bacon

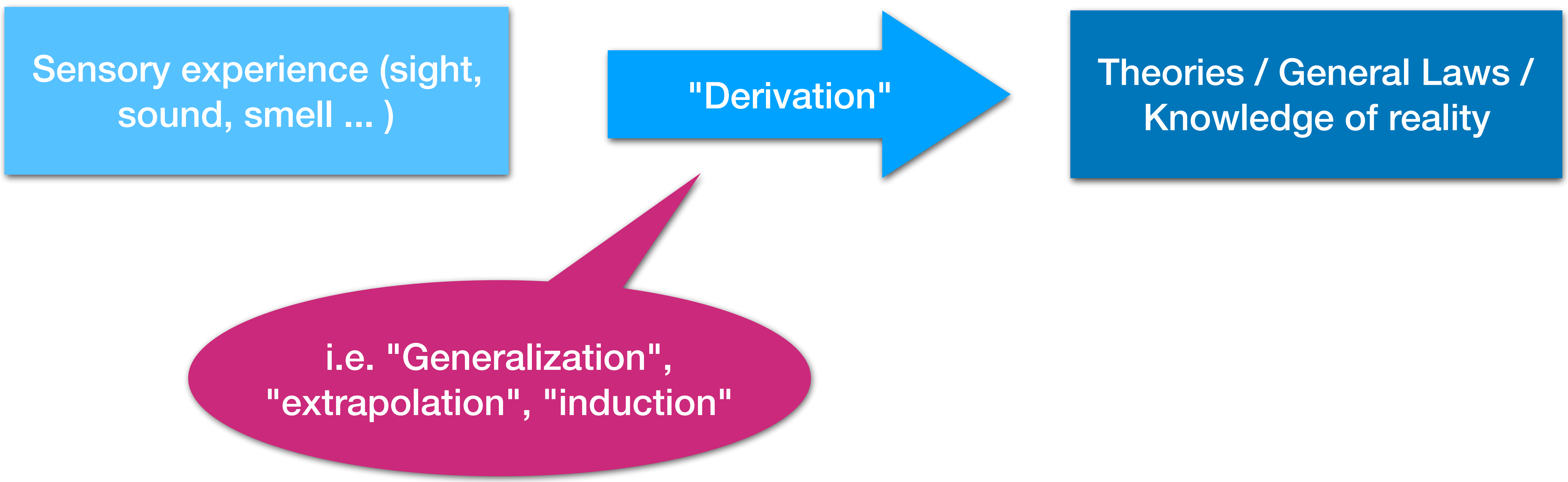


John Locke

David Hume

The Empiricists

- ***Where does knowledge come from?***
- The empiricists said it comes ***in through the senses***



Induction vs Deduction

- Given two statements X, Y we write $X \vdash Y$ if X ***entails*** Y
- Entailment is ***deductive***
- For example, if

G	= *All swans are white*
E_1	= *Swans in Vancouver are white*
E_2	= *Swans in Seattle are white*
E_3	= *Swans in London are white*
E_4	= *Swans in ...*

- $G \vdash (E_1 \wedge E_2 \wedge \ldots)$ is ***logically valid***

Induction vs Deduction

- However, $(E_1 \wedge E_2 \wedge \ldots) \vdash G$ is ***logically invalid***
- This is the problem of induction: ***How can we logically derive general laws G from evidence statements E?***
- Two proposed solutions:
  - Probabilistic solution
  - Popper's solution
- ... and 85 years of controversy

Induction vs Deduction

Popper, K. R. and Miller. D. (1983): 'A Proof of the Impossibility of Inductive Probability'. Nature, 302, pp. 687-688.

Jeffrey. R. C. (1984): 'The Impossibility of Inductive Probability'. Nature. 310, p. 433.

Popper, K. R. and Miller. D. (1984): 'The Impossibility of Inductive Probability', Nature, 310, p. 434.

Good, I. J. (1984): 'The Impossibility of Inductive Probability', Nature, 310, p. 434.

Levi, I. (1984): 'The Impossibility of Inductive Probability', Nature, 310, p. 433.

Gillies, D. (1986): 'In Defense of the Popper-Miller Argument', Philosophy of Science,53, pp. 110-113.

Redhead, M. L. G. (1985): 'On the Impossibility of Inductive Probability'. British Journal for the Philosophy of Science, 36, pp. 185-191

Gillies, D. (1986). In Defense of the Popper–Miller Argument. Philosophy of Science, 53(1), 110–113.

Levi, I. (1986). Probabilistic pettifoggery. Erkenntnis, 25(2), 133–140.

Good, I. J. (1987). A Reinstatement, in Response to Gillies, of Redhead's Argument in Support of Induction. Philosophy of Science, 54(3), 470–472.

Rodriguez, A. R. (1987). On Popper–Miller's proof of the impossibility of inductive probability. Erkenntnis, 27, 353–357.

Popper, K. R. & Miller, D. W. (1987). Why Probabilistic Support is Not Inductive. Philosophical Transactions of the Royal Society of London, A321, 569–596.

Townsend, B. (1989). Partly Deductive Support in the Popper-Miller Argument. Philosophy of Science, 56(3), 490–496.

Good, I. J. (1990). Discussion: A Suspicious Feature of the Popper/Miller Argument. Philosophy of Science, 57, 535–536.

Miller, D. W. (1990). Reply to Zwirn & Zwirn. Cahiers du CREA, 14, 149–153.

Chihara, C. S. & Gillies, D. A. (1988). An Interchange on the Popper–Miller Argument. Philosophical Studies, 54, 1–8.

Mura, A. (1990). When Probabilistic Support Is Inductive. Philosophy of Science, 57(2), 278–289.

Elby, A. (1994). Contentious Contents: For Inductive Probability. The British Journal for the Philosophy of Science, 45(1), 193–200.

Probabilistic solution

- Let G, E be as above, $P(G)$ be our prior belief in G, and $P(E)$ be the probability of the evidence
- Because $G \vdash E$, $P(E \mid G) = 1$
- Therefore:

$$P(G \mid E) = \frac{P(E \mid G)P(G)}{P(E)} = \frac{P(G)}{P(E)}$$

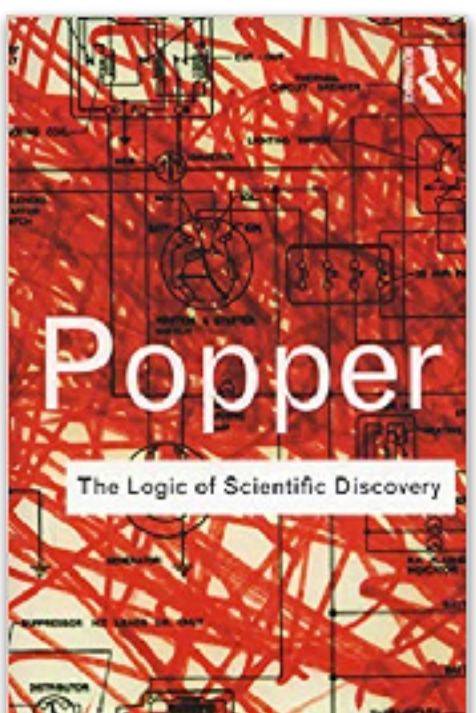
- And because $0 < P(E) < 1$

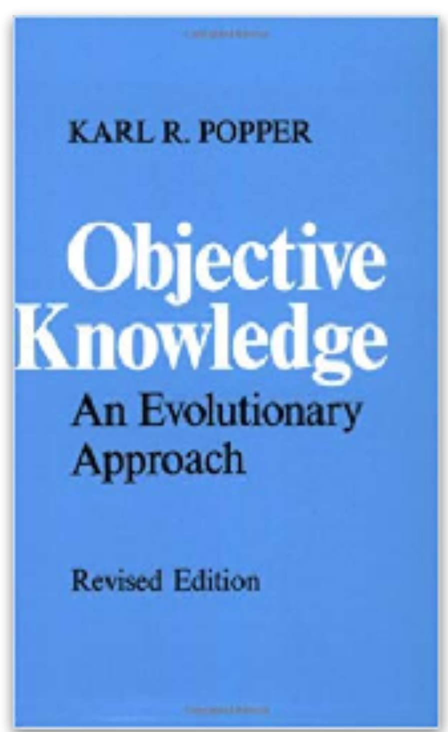
$$P(G \mid E) > P(G)$$

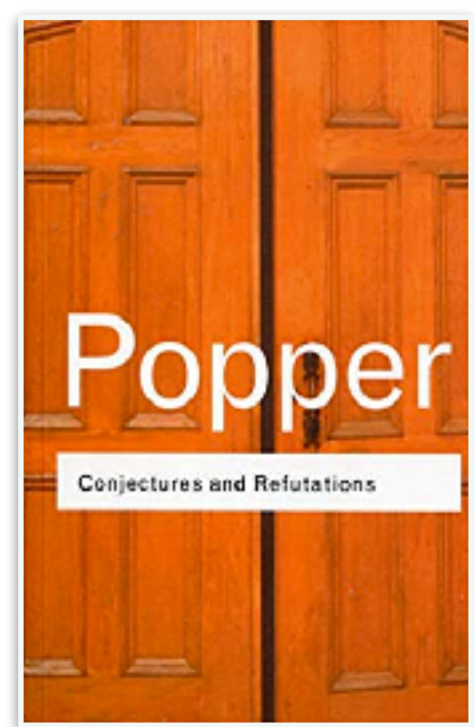
- Therefore the $P(G)$ increases with favorable evidence E

Popper's solution

- Realize that while $(E_1 \wedge E_2) \vdash G$ is invalid, $(E_1 \wedge \ldots \neg E_i \ldots) \vdash \neg G$ ***is valid***
- You can't induce truth ***but you can induce falsity***
- Therefore all our G's remain conjectural
- We can possess truth, though never with certainty







What I advocate (as it were, in the place of induction) ***is the admission of the fact that our hypotheses, our conjectures, are our intuitive guesses, results of our creative imagination****; results which, just because of their thrilling but dubious origin, ought to be submitted to the most grueling tests.*

–Popper, The Non-existence of Probabilistic Inductive Support

Who is more rational? The believer in generalizing induction, or the critic who tries hard to anticipate, and to eliminate, the mistakes that may lurk in his conjectures ...?

–Popper, The Non-existence of Probabilistic Inductive Support

Is it irrational to teach the distinction between truth and certainty? To say that we ought to search for truth, which we may discover, though we may never be certain of it?

–Popper, The Non-existence of Probabilistic Inductive Support

Conclusion

- Whatever your definition, an AGI will be an entity that can ***produce new knowledge***
  - Therefore we should take epistemology (knowledge about knowledge) seriously
- What are the limitations of the probability calculus?
- Could we train language models to maximize content rather than probability?
- Other fields are realizing they have to grapple with these questions, our field will too.

Other readings

AGI

1. *How Close Are We To Creating Artificial Intelligence?* David Deutsch, 2012 Aeon Essay
2. Chapter 7, *Beginning of Infinity: Explanations that Transform the World*. David Deutsch 2011
3. *NLPs Clever Hans Moment Has Arrived*, The Gradient, 2019
4. *A critique of pure learning and what artificial neural networks can learn from animal brains*, Nature, 2019

Epistemology

1. *Knowledge and the Mind Body Problem,* Popper 1994
2. Chapter 2, *Objective Knowledge*, Popper 1972
3. *The Blank Slate*, Steven Pinker, 2002

Induction

1. Chapter 3 and 7, *Fabric Of Reality*, David Deutsch 1997
2. Part 4, *The Non-existence of Probabilistic Inductive Support*, Popper 1985
3. Chapter 1, *Objective Knowledge*, Popper 1972
4. Chapter 2, *Realism and the Aim of Science*, Popper 1956

Content

1. Chapters 3 - 7, *The Logic of Scientific Discovery*, Popper 1934
2. Part 10, *Conjectures and Refutations*, Popper 1963
3. Chapter 2, *Objective Knowledge*, Popper 1972

Probability

1. Harvey Brown, *Aspects of Probabilistic Reasoning in Physics, Seminar on Probability* (Link is hard to find, see my website.)
2. *Diversity in interpretations of probability: implications for weather forecasting*, de Elía, Ramón; Laprise, René 2005.
3. Part II (in particular the Addendum of chapter 2) *Realism and the Aim of Science*, Popper 1992
4. Chapters 8 - 10, *The Logic of Scientific Discovery*, Popper 1934

Truth

1. Chapter 9, *Objective Knowledge*, Popper, 1972